Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 91573

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2014.

First Semester

Civil Engineering

MA 2111/MA 12/080030001 — MATHEMATICS – I

(Common to all Branches)

(Regulation 2008)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If λ be an eigenvalue of a non-singular matrix A, show that λ^{-1} is an eigenvalue of A^{-1}.

2. Find the eigenvalues of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

3. Find the centre and radius of the sphere $a(x^2 + y^2 + z^2) + 2ux + 2vy + 2wz + d = 0$.

4. Find the equation of the plane containing the line $\frac{x-1}{2} = \frac{y+1}{7} = \frac{z-2}{1}$ and parallel to the line $\frac{x}{1} = \frac{y-2}{2} = \frac{z-3}{3}$.

5. Find the envelope of the family of circles $(x-\alpha)^2 + y^2 = 4\alpha$, where α is the parameter.

6. Find the curvature of the curve $2x^2 + 2y^2 + 5x - 2y + 1 = 0$.

7. If $u = (x-y)^4 + (y-z)^4 + (z-x)^4$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

8. Find $\frac{\partial(x,y)}{\partial(r,\theta)}$, if $x = r\cos\theta$, $y = r\sin\theta$.

9. Evaluate $\int_0^4 \int_0^{x^2} e^{\frac{y}{x}} dy\, dx$.

10. Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z} dx\, dy\, dz$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. (8)

(ii) Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ and show that A satisfies the equation. Hence evaluate A^{-1}. (8)

Or

(b) Through an orthogonal transformation, reduce the quadratic form $x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 6x_3x_1$ to a canonical form. (16)

12. (a) (i) Find the centre and radius of the sphere whose equation is $x^2 + y^2 + z^2 - 2x - 4y - 6z - 2 = 0$. Show that the intersection of this sphere and the plane $x + 2y + 2z - 20 = 0$ is a circle whose centre is the point (2, 4, 5) and find the radius of the circle. (8)

(ii) Find the equation of the plane passing through the line of intersection of the planes $2x + y + 3z - 4 = 0$ and $4x - y + 5z - 7 = 0$ and is perpendicular to the plane $x + 3y - 4z + 6 = 0$. (8)

Or

(b) (i) Find the equation of the right circular cylinder of radius 2 and whose axis is the line $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-3}{2}$. (8)

(ii) Find the equation of the cone whose vertex is the origin and guiding curve is $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{9} = 1$, $x + y + z = 1$. (8)

13. (a) (i) Find the equation of the evolute of the parabola $y^2 = 4ax$. (8)

(ii) Find the equation of the circle of curvature at (c, c) on $xy = c^2$. (8)

Or

(b) (i) Show that the radius of curvature at any point of the catenary $y = c\cosh(x/c)$ is y^2/c. Also find ρ at $(0, c)$. (6)

(ii) Considering the evolute as the envelope of the normals, find the evolute of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$. (10)

14. (a) (i) If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$. (6)

(ii) Expand $e^x \log(1 + y)$ in powers of x and y upto terms of third degree. (10)

Or

(b) (i) A rectangular box, open at the top, is to have a volume of 32*cc*. Find the dimensions of the box, that requires the least material for its construction. (8)

(ii) Find $\frac{du}{dx}$, if $u = \sin(x^2 + y^2)$, where $a^2x^2 + b^2y^2 = c^2$. (8)

15. (a) Evaluate $\iint (x + y)^2 dx\, dy$ over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (16)

Or

(b) (i) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{(1-x^2-y^2)}} xyz\, dx\, dy\, dz$. (8)

(ii) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy$ by changing to polar coordinates. (8)
