Reg. No. :

# Question Paper Code : 41307

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Second Semester

Mechanical Engineering

MA 6251 – MATHEMATICS – II

(Common to Mechanical Engineering (Sandwich)/Aeronautical Engineering/ Agriculture Engineering/Automobile Engineering/Biomedical Engineering/ Civil Engineering/Computer Science and Engineering/Computer and Communication Engineering/Electrical and Electronics Engineering/Electronics and Communication Engineering /Electronics and Instrumentation Engineering/ Environmental Engineering/Geoinformatics Engineering/Industrial Engineering/ Industrial Engineering and Management/Instrumentation and Control Engineering/Manufacturing Engineering/Materials Science and Engineering/ Mechanical and Automation Engineering/Mechatronics Engineering/Medical Electronics/Petrochemical Engineering/Production Engineering/Robotics and Automation Engineering/(Common to all Branches except Marine Engg.)/Bio Technology/Chemical Engineering/Chemical and Electrochemical Engineering/ Fashion Technology/Food Technology/Handloom and Textile Technology/ Information Technology/Petrochemical Technology/Petroleum Engineering/ Pharmaceutical Technology/Plastic Technology/Polymer Technology/Textile Chemistry/Textile Technology/Textile Technology (Fashion Technology)

(Regulations 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions.

PART – A (10×2=20 Marks)

1. In what direction from (–1, 1, 2) is the directional derivative of $\phi = xy^2z^3$ a maximum ?

2. Find the value of 'a' for the vector $\vec{F} = (2x^2y + yz)\,\vec{i} + (xy^2 - xz^2)\,\vec{j} + (axyz - 2x^2y^2)\vec{k}$ to be solenoidal.

3. Find the complementary function $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = 0$ .

4. Write the general form of Cauchy's homogeneous linear equation.

5. If $f(t) = \begin{bmatrix} 3, 0 < t < 2 \\ -1, 2 < t < 4 \\ 0, t \geq 4 \end{bmatrix}$, find L[f(t)].

6. State and prove change of scale property.

7. Verify whether $w = (x^2 - y^2 - 2xy) + i x^2 - y^2 + 2xy)$ is an analytic function of $z = x + iy$.

8. Define conformal mapping.

9. Evaluate $\int_C (z^2 - z + 1)\, dz$ where C is the circle $|z| = 2$.

10. Write the Laurent's series expansion.

PART – B (5×16=80 Marks)

11. a) i) Prove that div grad $r^n = n(n+1)r^{n-2}$. (8)

ii) Find the work done in moving a particle in the force field $\vec{F} = 3x^2\,\vec{i} + (2xz - y)\,\vec{j} + z\,\vec{k}$ along the straight line from (0, 0, 0) to (2, 1, 3). (8)

(OR)

b) i) Evaluate $\oint_c \left[xy + x^2\right] dx + \left[x^2 + y^2\right] dy$ where c is the square formed by the lines $x = 1, x = -1, y = 1, y = -1$ using Green's theorem in the plane. (6)

ii) Verify Stoke's theorem for $\vec{F} = y^2\,\vec{i} + y\,\vec{j} - xz\,\vec{k}$ over the upper half of the sphere $x^2 + y^2 + z^2 = a^2, z \geq 0$. (10)

12. a) i) Solve : $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x \sin(\log x)$. (8)

ii) Solve by the method of variation of parameters $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1 + e^x}$. (8)

(OR)

b) i) Solve $(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x + 2)\frac{dy}{dx} - 36y = 3x^2 + 4x + 1$. (8)

ii) Solve the system of equations $\frac{dx}{dt} + 2y = -\sin t; \frac{dy}{dt} - 2x = \cos t$. (8)

13. a) i) Find the Laplace transform of $e^{-t} t^2 \sin 2t$. (8)

ii) Obtain the Laplace transform of the periodic saw-tooth wave function given by $f(t) = \frac{kt}{\omega}$ for $0 < t < \omega$, and $f(t + \omega) = f(t)$. (8)

(OR)

b) i) Using convolution theorem, find $L^{-1}\left\{\frac{1}{s^3(s+1)}\right\}$. (8)

ii) Solve $(D^2 - D - 2)\, y = 20 \sin 2t$ given that $y = -1$, $Dy = 2$ when $t = 0$ by using Laplace transform methods. (8)

14. a) i) Find the harmonic conjugate of the function $v(x, y) = e^x [x \sin y + y \cos y]$ if $f(z) = u + iv$. (8)

ii) Construct the analytic function $f(z) = u(r, \theta) + iv(r, \theta)$. Given that $u(r, \theta) = r^2 \cos 2\theta - r \cos\theta + 2$. (8)

(OR)

b) i) Find the image of the line $y = 3x + 1$ under the transformation $w = z^2$. (6)

ii) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. Hence find the image of $|z| < 1$. (10)

15. a) i) Evaluate $\int_C \frac{z-1}{(z+1)^2(z-2)}\, dz$ where C is the circle $|z - i| = 2$ by Cauchy's Integral Formula. (8)

ii) Find the Taylor's series expansion for the function $f(z) = \frac{1}{(1+z)^2}$ about $z = -i$. (8)

(OR)

b) i) Using Cauchy's residue theorem, evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)}\, dz$ where C is the circle $|z| = \frac{3}{2}$. (8)

ii) Prove that $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta} = \frac{2\pi}{\sqrt{3}}$ using contour integration. (8)