

Reg. No.:						
-----------	--	--	--	--	--	--

Question Paper Code: 41307

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Second Semester

Mechanical Engineering

MA 6251 - MATHEMATICS - II

(Common to Mechanical Engineering (Sandwich)/Aeronautical Engineering/
Agriculture Engineering/Automobile Engineering/Biomedical Engineering/
Civil Engineering/Computer Science and Engineering/Computer and
Communication Engineering/Electrical and Electronics Engineering/Electronics
and Communication Engineering / Electronics and Instrumentation Engineering/
Environmental Engineering/Geoinformatics Engineering/Industrial Engineering/
Industrial Engineering and Management/Instrumentation and Control
Engineering/Manufacturing Engineering/Materials Science and Engineering/
Mechanical and Automation Engineering/Mechatronics Engineering/Medical
Electronics/Petrochemical Engineering/Production Engineering/Robotics and
Automation Engineering/(Common to all Branches except Marine Engg.)/Bio
Technology/Chemical Engineering/Chemical and Electrochemical Engineering/

Fashion Technology/Food Technology/Handloom and Textile Technology/
Information Technology/Petrochemical Technology/Petroleum Engineering/
Pharmaceutical Technology/Plastic Technology/Polymer Technology/Textile
Chemistry/Textile Technology/Textile Technology (Fashion Technology)

(Regulations 2013)

Time: Three Hours

Maximum: 100 Marks

Answer ALL questions.

PART - A

 $(10\times2=20 \text{ Marks})$

- 1. In what direction from (-1, 1, 2) is the directional derivative of $\phi = xy^2z^3$ a maximum?
- 2. Find the value of 'a' for the vector $\vec{F} = (2x^2y + yz)\vec{i} + (xy^2 xz^2)\vec{j} + (axyz 2x^2y^2)\vec{k}$ to be solenoidal.
- 3. Find the complementary function $\frac{d^3y}{dx^3} 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} 2y = 0$.
- 4. Write the general form of Cauchy's homogeneous linear equation.



5. If
$$f(t) = \begin{bmatrix} 3, 0 < t < 2 \\ -1, 2 < t < 4 \\ 0, t \ge 4 \end{bmatrix}$$
, find L[f(t)].

- 6. State and prove change of scale property.
- 7. Verify whether $w = (x^2 y^2 2xy) + ix^2 y^2 + 2xy$ is an analytic function of z = x + iy.
- 8. Define conformal mapping.
- 9. Evaluate $\int_{C} (z^2 z + 1) dz$ where C is the circle |z| = 2.
- 10. Write the Laurent's series expansion.

$$PART - B$$

(5×16=80 Marks)

- 11. a) i) Prove that div grad $r^n = n(n+1)r^{n-2}$.
 - (8)
 - ii) Find the work done in moving a particle in the force field $\vec{F} = 3x^2 \vec{i} + (2xz y) \vec{j} + z \vec{k}$ along the straight line from (0, 0, 0) to (2, 1, 3). (8)
 - b) i) Evaluate $\oint [xy + x^2] dx + [x^2 + y^2] dy$ where c is the square formed by the lines x = 1, x = -1, y = 1, y = -1 using Green's theorem in the plane. (6)
 - ii) Verify Stoke's theorem for $\vec{F} = y^2 \vec{i} + y \vec{j} xz \vec{k}$ over the upper half of the sphere $x^2 + y^2 + z^2 = a^2$, $z \ge 0$. (10)
- 12. a) i) Solve: $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = \log x \sin(\log x)$. (8)
 - ii) Solve by the method of variation of parameters $\frac{d^2y}{dx^2} 3\frac{dy}{dx} + 2y = \frac{e^x}{1 + e^x}$. (8)
 - b) i) Solve $(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2)\frac{dy}{dx} 36y = 3x^2 + 4x + 1$. (8)
 - ii) Solve the system of equations $\frac{dx}{dt} + 2y = -\sin t; \frac{dy}{dt} 2x = \cos t.$ (8)



13. a) i) Find the Laplace transform of e^{-t} t² sin 2t.

- (8)
- ii) Obtain the Laplace transform of the periodic saw-tooth wave function given by $f(t) = \frac{kt}{\omega}$ for $0 < t < \omega$, and $f(t + \omega) = f(t)$. (8)
- b) i) Using convolution theorem, find $L^{-1}\left\{\frac{1}{s^3(s+1)}\right\}$. (8)
 - ii) Solve $(D^2 D 2)$ y = 20 sin2t given that y = -1, Dy = 2 when t = 0 by using Laplace transform methods. (8)
- 14. a) i) Find the harmonic conjugate of the function $v(x, y) = e^x [x \sin y + y \cos y]$ if f(z) = u + iv. (8)
 - ii) Construct the analytic function $f(z) = u(r, \theta) + iv(r, \theta)$. Given that $u(r, \theta) = r^2 \cos 2\theta r \cos \theta + 2$. (8)
 - b) i) Find the image of the line y = 3x + 1 under the transformation $w = z^2$. (6)
 - ii) Find the bilinear transformation which maps the points z = 1, i, -1 into the points w = i, 0, -i. Hence find the image of |z| < 1. (10)
- 15. a) i) Evaluate $\int_C \frac{z-1}{(z+1)^2(z-2)} dz$ where C is the circle |z-i|=2 by Cauchy's Integral Formula. (8)
 - ii) Find the Taylor's series expansion for the function $f(z) = \frac{1}{(1+z)^2}$ about z = -i. (8)
 - b) i) Using Cauchy's residue theorem, evaluate $C \int_{c} \frac{4-3z}{z(z-1)(z-2)} dz$ where C is the circle $|z| = \frac{3}{2}$. (8)
 - ii) Prove that $\int_{0}^{2\pi} \frac{d\theta}{2 + \cos \theta} = \frac{2\pi}{\sqrt{3}}$ using contour integration. (8)

- to at the deal that I replace beautiful and a to the St.
- may make the Laglace transfer of the period was attended to myelentra scales \mathcal{X} and the \mathcal{X}

(6)
$$(0) = \{ (a + b) \} \lim_{n \to \infty} a_n x + b = \frac{2\pi}{n} = (a) + a$$
(6)
$$(0) = \{ (a + b) \} \lim_{n \to \infty} a_n x + b = \frac{2\pi}{n} = (a) + a$$

- (6) if then convolution theorem, find (... $\left\{\frac{1}{a^*(a+1)}\right\}$.
- II) Subsection D = 21 y = 20 sings given that y = -1, Dy = 2 when t = 0 by using Laplace transform matheds.
- [4, 5) [1 Fold the barranetic conjugate of the function v is, y) = cf [s singer y comp].

 If f(x) = u + m.

 (5)
 - (i) Construct the analytic function f(x) = u(0, 0) + iv(x, 0), Given that

- by 3 Find the image of the line y=3s+1 under the transformation $w=x^2$. (6)
- ny reset etny tellinisary pasamatananakan withich magas dan pointu s = 1, i, =1 timo
 (10)

 The pointur w = 1, 0, = 1. Hence find the intrace of [4] < 2. (10)
- 16. 10 in Evaluation (3. +11 (a 2)) do where C is the state of -1 = 2 to Camby's

 [16. 10 in Evaluation (3. +11 (a 2)) do where C is the state of -1 = 2 to Camby's

 [16. 10 in Evaluation (3. +11 (a 2)) do where C is the state of -1 = 2 to Camby's

 [17. 10 in Evaluation (3. +11 (a 2)) do where C is the state of -1 = 2 to Camby's

 [18. 10 in Evaluation (3. +11 (a 2)) do where C is the state of -1 = 2 to Camby's

 [19. 11 (a 2) (a 2
- (8) thought the Transfer and the transfer that the foundation of the part and the first the foundation of the first the first
 - in a Union Constant and the thousand avalance $C = \frac{1-2a}{a(a-1)(a-2)}$ de where C is
- in Prace that \(\frac{dy}{2\sigma \cong \frac{2\pi}{12}} \) using contint integration (3)